

Example of finding total resistance and associated uncertainty

Find the total resistance for two resistors, R_1 and R_2 connected in (a) series, and (b) parallel.

$$R_1 = 50 \, \Omega \pm 5\%, \quad R_2 = 100 \, \Omega \pm 5\%$$

Note: Assuming that the 5% tolerance values for each resistor are not correlated, the uncertainty values can be treated as independent uncertainties, which means they can be added in quadrature. The more conservative approach is to treat these uncertainties as correlated, so that they always combine in the same direction (upper/lower bound method or worst-case scenario). For this example, the uncertainties will be treated as uncorrelated.

a) $R_T = R_1 + R_2 = (50 \, \Omega \pm 2.5 \, \Omega) + (100 \, \Omega \pm 5 \, \Omega) = 150 \, \Omega \pm 5.6 \, \Omega = 150 \, \Omega \pm 4\%$

$$\mathbf{R_T = (150 \pm 6)\Omega}$$

Note: The total resistance is larger than either of the individual resistors, but the relative uncertainty of R_T (4%) is smaller than the tolerance of either resistor in series (5%).

b) $1/R_T = 1/R_1 + 1/R_2 = (1/50 \pm 5\%) \, \Omega^{-1} + (1/100 \pm 5\%) \, \Omega^{-1}$

$$1/R_T = (0.02 \pm 0.001) \, \Omega^{-1} + (0.01 \pm 0.0005) \, \Omega^{-1} = (0.03 \pm 0.0011) \, \Omega^{-1} = 0.03 \, \Omega^{-1} \pm 3.7\%$$

$$\mathbf{R_T = 33.3 \, \Omega \pm 3.7\% = (33 \pm 1)\Omega}$$

Note: The total resistance is smaller than either of the individual resistors in parallel, and the relative uncertainty of R_T (4%) is also less than the tolerance of either resistor.